Analysis of the results using dimensionless numbers gives the calculational dependence

$$\frac{\alpha_m}{\alpha_a} = c\,\mathrm{Re}^n \mu^m. \tag{1}$$

The values of the constants c, n, and m are given in Table 1.

Thus, the experimental results obtained demonstrate that there is a significant change in the extent to which the gas velocity and the particle concentration affect the heat-transfer coefficient of a disperse current on passing from laminar to turbulent flow. In a number of cases this may be the main cause of the discrepancy between results obtained by different workers.

NOTATION

μ is the concentration of solid particles in air, kg/kg;
α_a is the heat-transfer coefficient of air, $W/m^2 \cdot °C$;
α_m is the heat-transfer coefficient of mixture of air and solid particles, $W/m^2 \cdot °C$;
Re_{cr_1} is the Reynolds number characterizing the change from laminar to transitional flow;
Re_{cr_2} is the Reynolds number characterizing the change from transitional to turbulent flow.

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HEAT AND MASS TRANSFER OF LARGE DROPLETS IN HIGHLY TURBULENT FLOWS

R. S. Tyul'panov

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A model is proposed for the heat and mass transfer of spherical bodies and large droplets in a strongly turbulent gas flow, when the scale of the turbulence is larger than the diameter of the body. Theoretical formulas are compared with experimental results.

The motion of a droplet in a gas flow in different kinds of technological equipment, including power installations, is accompanied by evaporation and by heat exchange with the surrounding gaseous medium. For large drops which are not involved in turbulent velocity pulsations, when the flow is characterized by the condition $L > d$ (L is the scale of the turbulence and d is the diameter of the body), the heat and mass transfer have certain specific properties.

In a number of works on the heat and mass transfer of a body in a gas flow, the processes appearing in the above conditions were found to be strongly influenced by the intensity of turbulence ε.

For a cylinder, the maximum value of Nu was observed for $L/d \simeq 1.6$, and on this basis a resonance theory of transfer was developed [1, 10].

Leningrad Technological Institute of the Cellulose and Paper Industry.

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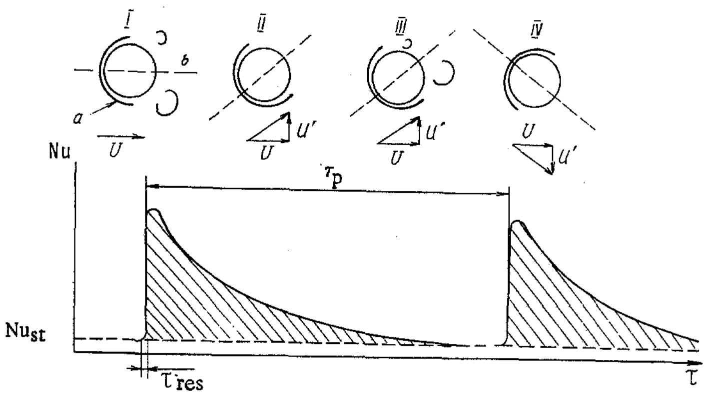

Fig. 1. Scheme of variation of Nu with change in flow velocity ($\varepsilon > 0.04$, $L/d > 1$, $40 < Re' < 1000$): a) boundary layer; b) back region; I) steady transfer in back region; II) change in velocity vector and boundary-layer breakaway; III) nonsteady transfer in back region; IV) change in velocity vector and boundary-layer breakaway; etc.

On the other hand, for the mass transfer of spherical bodies and droplets in an air flow, it was found that, if $L > d$ and $\varepsilon > 0.04$, the dependence of Nu on Re' in the range Re' = 40-1000 is comparatively simple [2]:

$$Nu = 2.8\sqrt{Re'}. \tag{1}$$

In this case no extremum was observed in the dependence of Nu on L/d, which disagrees with resonance transfer theory for a spherical body.

At the time, no explanation could be found for this lack of any significant effect of the mean relative flow velocity at large ε.

Physical models of the processes occurring for $L < d$, when turbulence of the flow has a marked effect on the heat and mass transfer of a body, have already been developed, and no further consideration will be given to this case.

I. It is now possible to formulate more clearly the physical picture of heat and mass transfer of a droplet for $L > d$. A droplet moving in a gas flow fairly rapidly acquires a velocity equal to the mean flow velocity, and under these condtions the role of turbulent pulsations may be taken to be determining. If the droplet nevertheless has a velocity different from the mean flow velocity, then for comparatively large drops a back eddy region is formed. For small turbulent pulsations of the velocity, the transfer process does not much depend on the velocity pulsations. If the turbulent pulsations of the flow velocity are large, so that $20 < du'/\nu < 1000$ for individual pulsations, the oscillating velocity leads to periodic breakaway of the boundary layer from the drop, emptying of the accumulated layer of transfer substance (heat or mass), especially in the back region; and displacement of the back region. In this case, the transfer process occurs throughout under unsteady conditions and considerably exceeds the transfer when a steady state has been established.

This mechanism of the effect of flow-velocity pulsations on the transfer process, shown schematically in Fig. 1, will be realized if

$$\tau_{res} < \tau_p \leqslant \tau_{st}. \tag{2}$$

Making some estimates, it appears that such a mechanism is a real possibility for the heat and mass transfer of a droplet in a turbulent flow. The formation of a boundary layer, up to its breakdown, occurs in a time $\tau_{res} = 0.2d/U$ [3], which is less than 10^{-5} sec for a drop of diameter d = 100 μ and flow velocity U = 20 m/sec. Therefore, for turbulent flow with mean pulsation 2000-5000 sec^{-1} in the usual technical apparatus, the characteristic pulsation time is $\tau_p = 0.2\text{-}0.5 \cdot 10^{-3}$ sec, i.e., much larger. In addition, the setting up of steady conditions of transfer, owing mainly to the presence of the back region, occurs in approximately the same time $\tau_{st} \simeq 10^{-3}$ sec, as indicated by calculations according to [4].

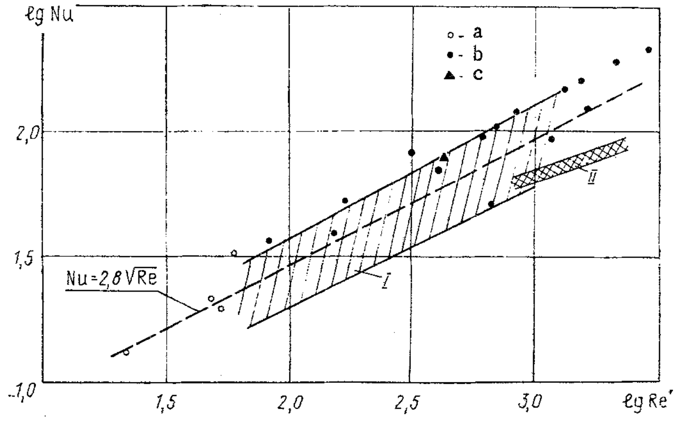

Fig. 2. Generalized dependence of log Nu on log Re' for spherical bodies ($\varepsilon > 0.04$, $L/d > 1$): I) region of experimental points for the mass transfer of a fixed ball of naphthalene [2]; a) evaporation of ethyl alcohol droplet moving in a gas flow; b) [12]; c) [11] (heat transfer of a sphere); II) region of experimental points for heat transfer in the presence of mass transfer [13].

For $\tau = 10^{-5}$ sec, the ratio of the values of Nu for nonsteady and steady transfer is $Nu/Nu_{st} = 10$.

The model developed in the present work for a sphere of mean relative velocity leads to the following polynomial, taking transfer under steady and nonsteady conditions into account:

$$\mathrm{Nu} = 2 + A_1 \mathrm{Re}^{0.5} \mathrm{Pr}^{0.33} + A_2 \mathrm{Re}'^{m} \left(\frac{L}{d} \right)^{n} \mathrm{Pr}^{k} \tag{3}$$

where A_1 and A_2 are numerical coefficients.

The first two terms in the right-hand side correspond to steady transfer, and the third corresponds to nonsteady transfer, of a spherical or almost-spherical body of droplet type.

It is evident from the analysis (Fig. 1) that in certain cases the last term may be dominant. For the example considered, the fraction of vapor (or heat) liberated in nonsteady transfer exceeds by a factor of 3-5 the fraction liberated in steady transfer of the drop in the gas flow.

II. Since the first papers on this subject were published, there has been a convincing accumulation of material confirming that turbulent pulsations have a pronounced effect on transfer processes under specific conditions, when the scale is larger than, or comparable with, the dimensions of the spherical body, over various ranges of the parameters [9, 12, 13]. Admittedly, there have been investigations in which no such strong effect of turbulence was observed for $L \sim d$ [15].

Formerly, many investigators tended to the view that the resonance theory developed for the heat transfer of a cylinder is also valid for the heat and mass transfer of a sphere. Recently, however, experiments [8, 9] have shown conclusively that the resonance theory cannot be applied to the sphere, since there is no maximum in the experimental values of Nu with increase in the scale of turbulence L, and the patterns of the flow past a sphere and a cylinder under highly turbulent conditions are significantly different [8, 14]. In [9, 13] it was shown experimentally that the important condition of increase in Nu for a sphere is that $L > d$, as our theory predicts. The conclusion that resonance theory is not valid for a sphere also appears in the review [5]. At the same time, the majority of experimental observations confirm the role of unsteady transfer in the total heat and mass transfer of spheres and droplets in a flow. Only when $L > d$ is the first part of Eq. (2) valid for turbulent pulsations:

$$\tau_{\mathrm{res}} \ll \tau_{\mathrm{p}}, \quad \text{since} \quad \tau_{\mathrm{p}} \sim L.$$

While there may be a dependence of Nu on L/d for a sphere, it is weak, and in a number of problems it may be neglected.

III. In considering the evaporation of drops over a restricted range of Re', ε, and L/d in a gaseous medium ($Pr \simeq 1$), it may be convenient to use an equation simpler than Eq. (3) – an equation of the type (1). To obtain such an equation, an analysis of the experimental data of [2] in the coordinates log Nu = f(log Re') is shown in Fig. 2, together with experimental results on the evaporation drops of ethyl alcohol obtained on apparatus similar to that described in [2] and also other experimental data on the heat transfer of a sphere [11, 12].

For the simultaneous heat and mass transfer characteristic of the evaporation of drops of a gas flow or a flame, discrepancies were obtained in the dependence Nu = f(Re) for different values of the injection parameter. With increase in the intensity of the turbulence $\varepsilon < 0.05$, these deviations considerably decrease and simultaneously there appears a sharp dependence of Nu on ε; for smaller ε, this dependence is very weak [13]. This completely agrees with the experimental results of [2]. Experimental results of [13] for the heat transfer of a sphere when $\varepsilon \geq 0.05$ are shown in Fig. 2 for various values of the injection parameter $\eta = M/\rho U = 0.1$-0.6, where M is the mass flow of material per unit surface of the sphere and ρU is the mass flow of washed gas.

Despite the scatter, it is very significant that, within ±25%, all the points corresponding to experiments carried out for Re' = 20-1000, i.e., under conditions such that the back region plays a very important role in transfer processes, and for $\varepsilon > 0.04$ and $L > d$ lie close to the straight line given by Eq. (1). Although Eq.(1) is a finite approximation, it suffices for calculations on the evaporation of drops in flows with large turbulent pulsations.

IV. From Eq. (1) it appears that if other flow and droplet parameters are held constant, the coefficient of heat or mass transfer is proportional to the mean-square pulsation velocity to the power 0.5. This figure was obtained experimentally; the model of nonsteady transfer that has been developed predicts only that it will be less than unity.

Since turbulent flow is characterized by a spectrum of velocity pulsations, while boundary-layer breakaway is only observed at a specific amplitude of pulsations, increase in the mean-square pulsational velocity will be accompanied by an increase in the fraction of pulsations that contribute to breakaway, determined by the spectral curve of the turbulence. For small wave number k, the spectrum corresponds to the law $E(k) \sim E^2k$ [10]. It is precisely that region of small k, where $E(k) \sim E^2k$ and where both the scale of the turbulence and the amplitude of the velocity pulsations are comparatively large, that is significant in the process of boundary-layer breakaway. Here E = Lu is the coefficient of turbulent viscosity.

We denote by u'_{br} the minimum velocity pulsation that leads to boundary-layer breakaway on a droplet or spherical particle. In accordance with the law assumed for small k, $u'_{br} \sim \sqrt{E(k)}$ is related to the frequency f_{br} corresponding to this velocity (or the wave number $k_{br} \sim f_{br}$) by the relation

$$f_{br} \sim \frac{u'^2_{br}}{E^2}. \tag{4}$$

The number of breakaways of the boundary layer from the drop, determining the magnitude of the nonsteady transfer, is

$$N_{br} = \int_{f_{br}}^{f_0} f\,df = \frac{1}{2}\left[f_0^2 - A\left(\frac{u'^2_{br}}{u^2L^2}\right)^2\right]. \tag{5}$$

The maximum frequency f_0 satisfies the condition $1/f_0 = \tau_{res}$; A = const; L, u'_{br} and f_0 are constants determined by the geometry of the system. Therefore, for the variation of the mean-square pulsational velocity u (or ε for a given value of the mean velocity U),

$$N_{br} = C - B/u^4, \tag{6}$$

where C = const and B = const.

Assuming that nonsteady transfer begins to appear at $\varepsilon > 0.04$, while $N_{br} = 0$ for $\varepsilon = 0.04$, the constant C can be determined from B and u when $\varepsilon = 0.04$ by the relation $C = B/u^4_{\varepsilon=0.04}$. Approximating N_{br} in powers of u gives

$$N_{br} = \frac{Cu^4 - B}{u^4} = \text{const}\, u^m. \tag{7}$$

Depending on the specific values of the constants C and B, which are always finite and positive, variation in N_{br}, and hence also in the magnitude of the nonsteady transfer, will accompany variation in u; however, it is evident from Eq. (7) that m should always be less than unity.

From what has been said it appears that Eq. (1) can only be used for certain characteristics of the frequency spectrum of the oscillations of a flow or jet corresponding to developed turbulence.

Recent experiments have shown that frequency spectra in flames are analogous to the spectra of turbulent pulsations in cold flows with developed turbulence, the scale being of the same order [7]. Further experimental confirmation is given in [6]. Therefore, Eq. (1) may also be applied to flames.

While the suggestions offered in the present work allow a number of features of the empirical formula in Eq. (1) to be explained, the quantitative theory of the heat and mass transfer of a sphere or droplet in a flow with large turbulent pulsations of the velocity stands in need of considerable further development.

NOTATION

d	is the body diameter;
L	is the integral scale of turbulence;
ε	is the intensity of turbulence;
U	is the mean velocity of flow relative to body;
u'	is the pulsational velocity of flow;
u	is the mean-square pulsational velocity;
ν	is the kinematic viscosity;
τ	is the time;
τ_{res}	is the time of residence (between formation and breakaway) of hydrodynamic boundary layer on drop;
τ_p	is the characteristic pulsation time of flow;
τ_{st}	is the time to establish steady transfer;
Nu	is the Nusselt number;
Re	is the Reynolds number;
Re'	is the Reynolds number based on mean-square pulsational velocity of flow and body diameter;
Pr	is the Prandtl number;
M	is the mass flow per unit surface area of sphere;
ρ	is the density;
u'_{br}	is the minimum pulsational velocity leading to boundary-layer breakaway;
f	is the pulsational frequency;
N_{br}	is the number of boundary-layer breakaways in unit time;
k	is the wave number;
$E(k)$	is the function describing pulsational energy spectrum;
f_0, f_{br}	are the maximum and minimum pulsational frequencies leading to boundary-layer breakaway.

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